Série N°1	Trigonométrie	1$^{\text{ère}}$ Spé

Correction

Exercice N°1 :

$$\alpha = 65° = 65 \times \frac{2\pi}{360} = \frac{13\pi}{36} \text{rad et} \qquad \beta = \frac{3\pi}{25}\ rad = \frac{\frac{3\pi}{25}360}{2\pi} = 21{,}6°$$

Exercice N°2 :

$-\frac{5789\pi}{13} + 22 \times 2\pi = \frac{9}{13}\pi$ or $\frac{9}{13}\pi \in]-\pi;\pi]$ donc $\frac{9}{13}\pi$ est la mesure principale de $(\vec{u};\vec{v})$.

Exercice N°3 :

On sait qu'un angle x dans l'intervalle $]-\frac{\pi}{2};\frac{\pi}{2}]$ vérifie $\sin x = -0{,}2$. Déterminer $\cos x$ et $\tan x$

$\cos^2 x + \sin^2 x = 1 \Leftrightarrow \cos^2 x + (-0{,}2)^2 = 1 \Leftrightarrow \cos^2 x = 1 - 0{,}04 \Leftrightarrow \cos x = \sqrt{0{,}96}$ ou $\cos x = -\sqrt{0{,}96}$ or $x \in]-\frac{\pi}{2};\frac{\pi}{2}]$ donc $\cos x \geq 0$ et donc $\cos x = \sqrt{0{,}96}$

De plus $\tan x = \frac{\sin x}{\cos x} = \frac{-0{,}2}{\sqrt{0{,}96}} = \frac{-0{,}2\sqrt{0{,}96}}{0{,}96} = -\frac{5}{24}\sqrt{0{,}96}$

Exercice N°4 :

1) $\cos\left(\frac{7\pi}{6}\right) = \cos\left(\pi + \frac{\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$, $\qquad \sin\left(-\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$,

$\cos\left(-\frac{5\pi}{3}\right) = \cos\left(2\pi - \frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

2) $\cos\frac{7\pi}{12} = \cos\left(\frac{\pi}{2} + \frac{\pi}{12}\right) = -\sin\left(\frac{\pi}{12}\right) = -\frac{\sqrt{2+\sqrt{3}}}{4}$, $\qquad$ et $\sin\frac{3\pi}{10} = \sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right) = \cos\left(\frac{\pi}{5}\right) = \frac{\sqrt{5}-1}{4}$.

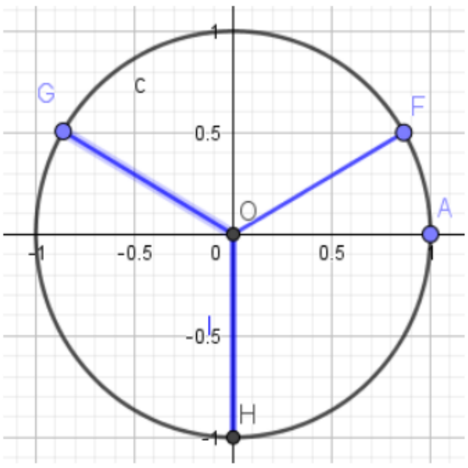
3) $\cos\left(\frac{2\pi}{3}\right) = \cos\left(\frac{5\pi}{3}\right) = -\frac{1}{2}$ et $\sin\left(\frac{5\pi}{3}\right) = \sin\left(\frac{-\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

Exercice N°5 :

Résoudre $\sin\left(2x + \frac{\pi}{6}\right) = sin\left(x + \frac{\pi}{3}\right)$ dans $\mathbb{R}$ puis dans $[0;2\pi]$

1) $\sin\left(2x + \frac{\pi}{6}\right) = sin\left(x + \frac{\pi}{3}\right) \Leftrightarrow \begin{cases} 2x + \frac{\pi}{6} = x + \frac{\pi}{3}\ [2\pi] \\ ou \\ 2x + \frac{\pi}{6} = \pi - \left(x + \frac{\pi}{3}\right)\ [2\pi] \end{cases} \Leftrightarrow \begin{cases} x = -\frac{\pi}{6} + \frac{\pi}{3}\ [2\pi] \\ ou \\ 3x = \pi - \frac{\pi}{3} - \frac{\pi}{6}\ [2\pi] \end{cases}$

$\Leftrightarrow \begin{cases} x = \frac{\pi}{6}\ [2\pi] \\ ou \\ 3x = \frac{\pi}{2}\ [2\pi] \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{6}\ [2\pi] \\ ou \\ x = \frac{\pi}{6}\ [\frac{2\pi}{3}] \end{cases}$



Solutions dans $[0;2\pi]$, la première équation me donne $\frac{\pi}{6}$ et la seconde $\frac{\pi}{6}, \frac{\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6}$ et $\frac{5\pi}{6} + \frac{2\pi}{3} = \frac{9\pi}{6} = \frac{3\pi}{2}$

$S = \left\{\frac{\pi}{6};\frac{5\pi}{6};\frac{3\pi}{2}\right\}$